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Resumos

TWISTED COHOMOLOGICAL EQUATION SOLUTION PROPERTIES

AMANDA DE LIMA AND DANIEL SMANIA

We are going to consider a family of maps $t \in (-\delta, \delta) \mapsto f_t \in C^1(S^1)$. If f_0 is an expanding map, then there is δ_0 such that for all $t \in (-\delta_0, \delta_0)$, f_t is also an expanding map and there is a homeomorphism h_t such that $h_t \circ f_0(x) = f_t \circ h_t(x)$. Differentiating this equation with respect to t , we obtain

$$v_t(y) = \alpha_t(f_t(y)) - \partial_x f_t(y) \alpha_t(y),$$

where $\alpha_t(y) := (\partial_t h_t) \circ h_t^{-1}(y_t)$ and $v_t(x) := \partial_t f_t(x)$. Fixing t , we have the *twisted cohomological equation*

$$(1) \quad v(y) = \alpha(f(y)) - Df(y)\alpha(y).$$

There exists a unique bounded function satisfying 1 and this function is given by

$$(2) \quad \alpha(x) = - \sum_{n=1}^{\infty} \frac{v(f^{n-1}(x))}{Df^n(x)}.$$

Let $f \in C^{2+\varepsilon}(S^1)$ be an expanding map and $v : S^1 \rightarrow R$ a periodic function of class $C^{1+\varepsilon}$. Our goal is to study smoothness properties the function α defined by (2). For example, the following theorem

Theorem 1: One of the following statements holds:

- (i) α is of class $C^{1+\varepsilon}$
- (ii) α is nowhere differentiable.

And a Central Limit Theorem-type

Theorem 2: Suppose that α is nowhere differentiable. Then there exists $\sigma > 0$ such that

$$\lim_{h \rightarrow 0} \mu \left\{ x : \frac{\alpha(x+h) - \alpha(x)}{h\sqrt{\ln h}} \leq y \right\} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{t^2}{2\sigma^2}} dt,$$

where μ is the absolutely continuous invariant measure with respect to the Lebesgue measure.

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TLONG-TIME DYNAMICS OF AN EXTENSIBLE PLATE EQUATION WITH THERMAL MEMORY

ALISSON RAFAEL AGUIAR BARBOSA AND MA TO FU

This is my abstract.

This work is concerned with long-time dynamics of solutions of extensible plate equations with thermal memory. It corresponds to a model of thermoelasticity based on a theory of non-Fourier heat flux. By considering the case where rotational inertia is present we show that the thermal dissipation is sufficient to stabilize the system exponentially and guarantee the existence of a finite-dimensional global attractor. In addition the existence of an exponential attractor and some further properties are also considered. Our results complements several existing results.

Key-words: Partial differential equations, thermoelasticity, extensible plates, thermal memory, non-Fourier heat flux, global attractor, exponential attractors.

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ANALYTIC REGULARITY FOR STRUCTURES OF CORANK ONE

ERIK FERNANDO DE AMORIM AND SERGIO LUIS ZANI

The goal of this work is to study conditions that characterize local and global analytic hypoellipticity of a locally integrable system of n linear partial differential equations, with real analytic coefficients, when the underlying manifold has dimension $n + 1$. Our main references are [1] and [2]. Our work was aimed at studying those and the proofs of their results in detail.

The general theory regards systems of first-order linear PDE as involutive structures defined on the tangent bundle of the manifold. This treatment can be found in the book [3], based on which we have also studied a modern proof of the main result of the theory: the Baouendi-Treves approximation formula. This formula gives a way to approximate any continuous solution of the structure by polynomials evaluated at the so called *first integrals* of the structure: continuous solutions whose differentials locally span the cotangent subbundle associated to it.

The article [1] was a precursor of the formula that bears the name of its authors. Here the case of locally integrable analytic structures of corank 1 is considered, and they give necessary and sufficient conditions for local analytic hypoellipticity, namely, that the first integrals be open mappings at the point of interest. This result was then used in [2] to prove a global characterization of the same property, in terms of the level sets of the local first integrals.

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SYMPLECTIC SINGULARITIES OF VARIETIES: THE METHOD OF ALGEBRAIC RESTRICTIONS

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We study germs of singular varieties in a symplectic space. In [A1], V Arnol'd discovered so called ghost symplectic invariants which are induced purely by singularity. We introduce algebraic restrictions of differential forms to singular varieties and show that this ghost is exactly the invariants of the algebraic restriction of the symplectic form. This follows from our generalization of Darboux-Givental' theorem from non-singular submanifolds to arbitrary quasi-homogeneous varieties in a symplectic space. Using algebraic restrictions we introduce new symplectic invariants. We prove that a quasi-homogeneous variety N is contained in a non-singular Lagrangian submanifold if and only if the algebraic restriction of the symplectic form to N vanishes. We give a complete solutions of symplectic classification problem for the classical A,D,E singularities of curves.

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GENERALIZED STRING LINKS OVER SURFACES

JULIANA R. THEODORO DE LIMA AND DENISE DE MATTOS

In this work we will talk about a presentation for this group and a possible order for it.

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SOME PROPERTIES OF THE GEOMETRIC LORENZ FLOW

JOSÉ HUMBERTO BRAVO VIDARTE AND DANIEL SMANIA BRANDÃO

In 1963, the meteorologist E.N. Lorenz published the paper *Deterministic Nonperiodic Flow* [3], introducing a quite simple system of differential equations that has unpredictable behavior.

$$(1) \quad (\dot{x}, \dot{y}, \dot{z}) = (\sigma(y - x), \rho x - y - xz, -\beta z + xy).$$

Setting the parameters (σ, ρ, β) at $(10, 20, \frac{8}{3})$, Lorenz numerically found solutions which remains bounded in the future. The solutions seems to wind around a pair of points, alternating on which point they encircle. This is the first important fact about the Lorenz systems: All nonequilibrium solutions tend to the same complicated set, the so-called *Lorenz Attractor*. Another fact that was noted by Lorenz was that the *Lorenz Attractor* has *sensitivity to initial conditions (the butterfly effect)*. No matter how close two solutions start, they will have a quite different behaviour in the future.

A rigorous mathematical study of the Lorenz equations was out of reach until a few years ago. The first mathematically important contribution to understand such “chaotic” dynamic was made independently by [2], and Afraimovich, Bykov and Shil’nikov [1]. They introduced a Lorenz Geometrical Model, a specially crafted type of flows that share many features with the Lorenz flow, and yet it was easier to study in a rigorous way. They have proven the existence of a well-defined attractor for the Geometrical Model.

In this lecture, we will present the definition of The Geometrical Model due to Guckenheimer [2] called, **The Geometric Lorenz Flow** and some properties of this flow.

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ON IMPULSIVE SEMIDYNAMICAL SYSTEMS: MINIMAL, RECURRENT AND ALMOST PERIODIC MOTIONS.

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This work concerns results about minimal, recurrent and almost periodic motions in impulsive semidynamical systems. First, we investigate general properties of minimal sets. In the sequel, we study some relations among minimal, recurrent and almost periodic motions. Some important results from the classical dynamical systems theory are generalized to the impulsive case, as Birkhoff's theorem for instance.

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HELIX SURFACES IN THE SPECIAL LINEAR GROUP

A. PASSOS PASSAMANI AND I. I. ONNIS

This is my abstract.

In the last years, much work has been done to study surfaces whose unit normal vector field forms a constant angle with a fixed field of directions of the ambient space. These surfaces are called *helix surfaces* or *constant angle surfaces* and they have been studied in most of the 3-dimensional geometries.

In this presentation we characterize the helix surfaces in the homogeneous 3-manifold given by the special linear group $SL(2, \mathbb{R})$ endowed with a suitable 1-parameter family g_τ of metrics. In particular, we give an explicit local description of these surfaces in terms of a suitable curve and a 1-parameter family of isometries of $SL(2, \mathbb{R})$.

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**PULLBACK ATTRACTORS FOR A SEMILINEAR WAVE
EQUATION ON TIME-VARYING DOMAINS**

CHRISTIAN M. S. CHUÑO, MA TO FU

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**C^∞ -SOLVABILITY NEAR THE CHARACTERISTIC SET
FOR A CLASS OF PLANAR COMPLEX VECTOR
FIELDS OF INFINITE TYPE**

WANDERLEY APARECIDO CERNIAUSKAS, PAULO LEANDRO DATTORI DA SILVA
AND ADALBERTO PANOBIANCO BERGAMASCO

We consider vector fields of the form

$$L = \partial/\partial t + (x^n a_0(x) + ix^m b_0(x))\partial/\partial x,$$

defined on $A_\epsilon = (-\epsilon, \epsilon) \times S^1$, $\epsilon > 0$, $2 \leq \min\{n, m\}$, where a and b are C^∞ real-valued functions in $(-\epsilon, \epsilon)$. Let $p \in C^\infty(A_\epsilon)$, we look for C^∞ solutions of the equation $Lu = pu + f$ in a full neighborhood of the characteristic set $\Sigma \doteq \{0\} \times S^1$. It is shown that the interplay between the order of vanishing of the functions a and b at $x = 0$ influences the C^∞ -solvability at Σ .

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NIELSEN'S ROOT THEORY FOR PROPER MAPS

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Let (X, \mathfrak{A}) be a connected topological n -manifold, (Y, σ) a well connected space, i.e., a connected, Hausdorff, locally path connected and semilocally simply connected topological space that is n -euclidean around a point $y_0 \in Y$ and $f : X \rightarrow Y$ a continuous map.

In this work we study a generalization of Nielsen's Root Theory, a branch of Algebraic Topology that studies some aspects of the pre-image of continuous maps from a compact, orientable and connected topological n -manifolds into a topological space that is well connected and n -euclidean around one of its points, $y_0 \in Y$. Some of these aspects are, for instance, the cardinality of the set of roots, the minimum number of roots among all the maps homotopic to f and whether there exists a map $g : X \rightarrow Y$ that accomplishes this minimum number or not. We will present the basic concepts concerning to this theory and the main theorems obtained in a more general case, when we allow the manifold (X, \mathfrak{A}) to be non-orientable and non compact, restraining ourselves to the study of proper maps. This study is based on Robin Brooks' paper, [1].

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NASH TRANSFORMATION OF DETERMINANTAL VARIETIES WITH ISOLATED SINGULARITIES

NANCY CHACHAPOYAS S. AND MARIA RUAS AND JEAN-PAUL BRASSELET

In this work, we study the Nash transformation of codimension two determinantal varieties with isolated singularities. This happens when the determinantal variety is a curve in \mathbb{C}^3 , a surface in \mathbb{C}^4 , a 3-dimensional variety in \mathbb{C}^5 or a 4-dimensional variety in \mathbb{C}^6 . We use the Nash Transformation to study the Poincaré Hopf index of differential forms defined on the variety.

When the determinantal variety X is a curve in \mathbb{C}^3 , the iterated Nash transformation is a resolution of the singularities of X . The results are exemplified using the classification of Anne Frühbis-Krüger and Alexander Neumer, [1, 2].

We also study the Nash transformation of families of determinantal curves.

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A SURVEY OF THE COHOMOLOGICAL DEGREE OF EQUIVARIANT MAPS

NORBIL CORDOVA, DENISE DE MATTOS, AND EDIVALDO DOS SANTOS

The first result in degree theory for equivariant maps was the famous Borsuk-Ulam theorem which states that the degree of an odd map of a finite-dimensional sphere into itself is odd [1]. The oddness presents the simplest example of equivariance with respect to $\mathbb{Z}_2 = \{I, -I\}$. This theorem was generalized by Krasnoselskii [2] substituting the first sphere (domain) by a topological manifold and the group \mathbb{Z}_2 by a finite group. Although, we can see that degree theory only depends of the (co)homology theory, for this reason, one of our results is to generalize the Krasnoselskii's theorem for spaces which reflect many of the local and global (co)homology properties of the topological manifolds. With this result we can calculate the existence of equivariant maps for free actions. Also we present another results of this theory.

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